

# Flow Control for Relay Traffic in Cognitive Cooperative Random Access

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## Abstract

This paper addresses the idea of flow control in a two-user cognitive cooperative system in which the secondary (cognitive) user is expected to cooperate with the primary user during the time slots in which they are not operating simultaneously. This type of network layer cooperation is in contrast with the traditional notion of cognitive radio, in which the secondary user is required to relinquish the channel as soon as the primary is detected. In the system under study, the primary user transmits whenever it has packets to transmit, while the secondary user bases its transmission decisions on the primary user's queue state. Additionally, the secondary user is equipped with a flow controller that controls the amount of cooperation it provides to the primary user. We characterize the stable-throughput region by taking into account the compound effects of multi-packet reception as well as the cooperative relaying capability of the cognitive user.

## I. INTRODUCTION

The traditional paradigm of cognitive radio communications allows for limited coexistence of users with different priorities in the same channel. The higher priority users, usually called primary (or, licensed, in commercial parlance), are allowed to access the spectrum at any time, while the secondary (lower priority) users are usually required to transmit opportunistically by taking advantage of the idle periods of the primary nodes. In contrast, this work addresses a class of cognitive shared channels, in which users of varying priority are allowed to coexist and transmit in the presence of one another. This model is of particular interest to the Department of Defense (DoD) spectrum management community. Our focus here is on the rate measure called stable throughput (packets/slot) [1] that is especially meaningful and relevant for wireless networks. It is defined for users that are not backlogged and receive bursty traffic, which is queued up at their buffers while awaiting transmission, and therefore requires the queues to be stable. The exact definition of stability used in this paper is provided later in Section II.

Earlier work by us [1], [2] as well as others [3], focused on the aspect of opportunistic cooperation at the network layer, which takes advantage of the broadcast nature of the wireless medium. In these works, during the time slots in which the secondary user is idle, it has a chance to capture the primary's packets and then cooperatively relay those packet to the intended destination. In such cognitive cooperative systems, especially in scenarios where the direct channel on the primary link is weaker than the link from the primary to the secondary transmitter, having packets relayed by the secondary would help to empty the primary queue, thus creating better transmitting opportunities for the secondary, and increasing the stable throughput of both the primary as well as the secondary node as compared to the non-cooperation case.

The main contribution of this work is that, we have introduced the notion of flow control at the secondary (relay) node. Specifically, the secondary user is equipped with a flow controller that controls the amount of cooperation it is willing to provide, by regulating the endogenous arrivals from the primary user so that all the queues in the network remain stable. By characterizing the optimal operation of the flow controller, we analyze the conditions under which the cognitive system should operate as a non-cooperative system, a full cooperative system, or as a partial cooperative system, so as to maximize the stable throughput region.

Furthermore, we obtain the exact characterization of the stable throughput region for the system with the flow controller, as shown in Figure 1. The characterization of the stability region is known to be challenging

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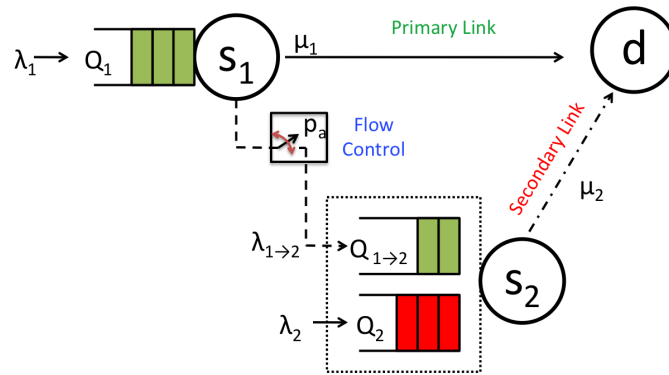


Fig. 1. Network Model for the Cognitive Cooperative System

in such multi-access systems, because the queues of the users are coupled i.e., the service process of a queue depends on the status of the other queues. We intend to utilize the stochastic dominance technique, which is described in Section III, in order to bypass this difficulty.

The remainder of the paper is organized as follows. In Section II, we present the system model that is used in the paper and detail the assumptions that are made. In Section III, we analyze the stability of the cognitive cooperative system with the flow controller, and derive the complete stable-throughput region based on the idea of stochastic dominance. Finally, we present our conclusions in Section IV.

## II. SYSTEM MODEL

In this work, we consider the basic cognitive cooperative network with two source nodes ( $s_1$  and  $s_2$ ) and one destination node ( $d$ ) as shown in Figure 1. Time is slotted and packets arrive at each source node independently according to a Bernoulli process with mean arrival rates of  $\lambda_i, i = 1, 2$  (packets/slot) in each time slot. Node  $s_1$  is the primary, or higher priority, user; it transmits a packet whenever its queue is non-empty, independently of the actions of the secondary node  $s_2$ . If the packet transmitted by  $s_1$  is decoded successfully by destination  $d$ , the packet exits the network and therefore will be removed from the primary queue. Node  $s_2$ , being the secondary or the cognitive node, must act in a way that depends on the actions of the primary node  $s_1$ . Specifically, node  $s_2$  observes the queue length  $Q_1$  at  $s_1$  and if  $Q_1 = 0$ , then  $s_2$  will transmit with probability 1 if its own queue  $Q_2$  is non-empty. Otherwise, if  $Q_1 \neq 0$ , which means that node  $s_1$  will transmit in the time-slot,  $s_2$  will transmit with probability  $p$ , resulting in concurrent transmissions. The destination node  $d$  is assumed to be equipped with multi-user detection [4] so that it can simultaneously decode packets from concurrent transmissions, albeit with a lesser probability than in the case of single transmissions. This is captured in the multi packet reception (MPR) channel model, which is described in detail in Section II-A.

The overall cooperation strategy is as follows: in the time slots that node  $s_2$  does not transmit, it has a chance to capture those packets from  $s_1$  that are not successfully decoded by the destination node  $d$ . Specifically, node  $s_2$  is equipped with a flow controller that regulates the rate with which these packets are decoded and stored in  $s_2$ 's buffer, by accepting incoming packets with probability  $p_a$ . Once a packet is received past the flow controller, it is  $s_2$ 's responsibility to relay this packet to the destination, thereby, allowing the packet to be removed from the primary queue. By controlling the value of  $p_a$ , node  $s_2$  can regulate the amount of cooperation it is willing to provide. In this paper, we analyze the landscape of  $p_a$  that maximizes the overall stable throughput region of the cognitive cooperative system. One can imagine that node  $s_2$  maintains separate virtual queues for these endogenous packets from  $s_1$  as well as its own exogenous packets that arrive at  $s_2$  with an average arrival rate of  $\lambda_2$  (packets/slot). However it has been shown in [3]

that the choice of packet transmission at the secondary node does not affect the stable throughput region, therefore we can let  $s_2$  maintain a single queue that is destined for  $d$ .

Following are the assumptions made in the paper, in order to facilitate the system model.

- The transmission of one packet takes a duration of exactly one time slot, and packets awaiting transmission by each source  $s_i$  are stored in infinite-sized buffers.
- Receivers are equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time. Such an MPR model has been studied extensively, and is similar to the model specified in [4]–[6].
- Nodes cannot transmit and receive at the same time. These transmission constraints are common in network systems where nodes are equipped with single transceivers with omni-directional antennas, that can only transmit one packet at a time, although, as technology advances, such restrictions can be relaxed.
- Positive acknowledgments (ACKs) from the receivers are assumed to be broadcast instantaneously and without error on a separate channel with negligible bandwidth. Therefore, the overhead due to acknowledgments is not considered in this analysis. This simplification can be overcome by a detailed analysis of the overhead caused by a realistic control-message exchange, but such an analysis would not contribute toward the main goal of our paper.
- The secondary user knows instantaneously whether or not the primary user's queue is empty. This can also be achieved via a channel sensing mechanism.

#### A. Channel Model

The MPR channel model can be described as follows. Given a set  $\mathcal{M}$  of source users that transmit simultaneously, the probability that a destination node  $n$  decodes a packet from a source  $m$ , where  $m \in \mathcal{M}$ , is given by:

$$q_{m|\mathcal{M}}^{(n)} = \mathbf{P} [\text{packet from node } m \text{ is received at node } n \mid \text{users in node set } \mathcal{M} \text{ transmit}].$$

In the two-user cognitive network shown in Figure 1, we are specifically interested in the following reception probabilities:

$$q_{1|1}^{(d)}, q_{2|2}^{(d)}, q_{1|1,2}^{(d)}, q_{2|1,2}^{(d)}, q_{1|1}^{(s_2)}.$$

It should be noted that, reflecting the behavior of real channels, the probability that a packet transmitted by a source  $s_i$  is decoded by the destination  $d$  given that only  $s_i$  transmits is larger than the corresponding probability given both  $s_1$  and  $s_2$  transmit simultaneously, i.e.,  $q_{1|1}^{(d)} \geq q_{1|1,2}^{(d)}$ ,  $q_{2|2}^{(d)} \geq q_{2|1,2}^{(d)}$ , etc.

The MPR channel model used in this paper is a generalized form of the packet-erasure model, which is known to capture reasonably well the behavior of the wireless channel. The reception probabilities  $q_{m|\mathcal{M}}^{(n)}$  that are part of the MPR model are not arbitrary quantities, but can, in fact, be derived based on the effects of fading, attenuation and interference at the receivers.

In wireless environments, if the receiver is equipped with multiple matched filters and treats interference as noise, the packet error probability can be maintained at an acceptable level, if the received signal-to-interference and noise ratio (SINR) exceeds a certain threshold. Therefore, one can compute the reception probabilities based on the probability  $q = \mathbf{P}[SINR > \theta]$ .

#### B. Queue Stability

Let us denote by  $Q_i^t$  the queue length at node  $s_i$  in time slot  $t$ . Then,  $Q_i^t$  evolves according to,

$$Q_i^{t+1} = [Q_i^t - Y_i^t]^+ + X_i^t,$$

where,  $Y_i^t$  is the number of departures at  $s_i$  in slot  $t$ ,  $X_i^t$  denotes the number of arrivals, and  $[x]^+ = \max(0, x)$ . Based on the definition in [7], a queue is said to be stable if

$$\lim_{t \rightarrow \infty} \mathbf{P}[Q_i^t < x] = F(x) \quad \text{and} \quad \lim_{x \rightarrow \infty} F(x) = 1.$$

For queues where the arrival and service processes are jointly strictly stationary and ergodic, Loynes' theorem [8] states that the queue at node  $s_i$  is stable if and only if the average arrival rate  $\lambda_i$  is strictly less than the average service rate denoted by  $\mu_i$ , i.e.,  $\lambda_i < \mu_i$ . If  $\lambda_i > \mu_i$ , the queue is unstable. We use this definition of stability although there exist several similar variations of the stability notion, which in many cases are equivalent.

A system is said to be stable if and only if all queues in the system are stable. The stable-throughput region of the system is defined as the set of arrival rates  $\lambda_i, i = 1, 2$ , for which the system is stable.

### III. STABILITY ANALYSIS

In this section, we derive the stable-throughput region of the cognitive cooperative network system  $\mathcal{S}$  described in Figure 1. Based on the system model described in Section II, the average service rates seen by  $s_1$  and  $s_2$  are given by

$$\begin{aligned} \mu_1 &= q_{1|1}^{(d)} \mathbf{P}[Q_2 = 0] + q_{1|1,2}^{(d)} \mathbf{P}[Q_2 \neq 0]p + q_{1|1}^{(d)} \mathbf{P}[Q_2 \neq 0](1-p), \\ \mu_2 &= q_{2|2}^{(d)} \mathbf{P}[Q_1 = 0] + q_{2|1,2}^{(d)} \mathbf{P}[Q_1 \neq 0]p. \end{aligned}$$

Since the mean service rates at  $s_1$  and  $s_2$  (i.e.,  $\mu_1$  and  $\mu_2$ ) depend on each other's queue size, these queues are called *interacting*. It is well known that the analysis of interacting queues is intractable; consequently the rates of the individual departure processes cannot be computed directly. In order to bypass this problem, we utilize the idea of stochastic dominance [7], which has been employed before to analyze interacting queues.

We first construct an appropriate *dominant system*, which is a modification of the original system, that ensures that the queue sizes in the dominant system are, at all times, at least as large as those of the original system. Thus, the stability region of the new system "inner bounds" that of the original system. Furthermore, in the new system the queues are decoupled (and consequently not interacting anymore), thereby permitting the characterization of the stability region. Second, we prove that the dominant system and the original system behave identically (i.e., are indistinguishable) at the boundary of the stability region, thereby causing the inner bound to coincide with the stability region of the original system.

Let  $\mathcal{S}'$  be the corresponding dominant system for the original system  $\mathcal{S}$ . In the case of the dominant system,

- If  $Q_2 = 0$ , node  $s_2$  transmits a dummy packet with probability 1, when  $Q_1 = 0$
- If  $Q_2 = 0$ , node  $s_2$  transmits a dummy packet with probability  $p$ , when  $Q_1 \neq 0$ ,

i.e., user  $s_2$  behaves as if its queues were never empty, except it transmits dummy packets when no real packets are in its queue. This achieves a constant average service rate to user  $s_1$  regardless of the state of  $s_2$ 's queue; consequently the two queues are now decoupled. All the other assumptions including channel models, arrival and reception processes remain unaltered.

#### A. Analysis at the Primary Node $s_1$

In the dominant system  $\mathcal{S}'$ , just as in the original system  $\mathcal{S}$ , the arrival rate at  $s_1$  is given by  $\lambda_1$ . The service process at  $s_1$  depends on which of two possible actions occurs: (i) if node  $s_2$  transmits along with  $s_1$  (which happens with probability  $p$ ), then the service rate seen by  $s_1$  is  $q_{1|1,2}^{(d)}$ ; (ii) if node  $s_2$  remains idle when  $s_1$  transmits, any packet that is successful at either  $d$  or  $s_2$  is dropped from  $Q_1$ , which happens with probability  $(1-p)$ .

The service rate that  $s_1$  receives in case (ii) can be computed as follows: when  $s_1$  transmits a packet, the probability that the packet is not successfully decoded by  $s_2$  is given by  $(1 - q_{1|1}^{(s_2)} p_a)$ , where  $p_a$  is the probability that the packet is allowed to reach  $s_2$  by the flow controller. The probability that the packet is not successful at either  $s_2$  or  $d$  can be computed as  $(1 - q_{1|1}^{(s_2)} p_a)(1 - q_{1|1}^{(d)})$ . Therefore, the service rate that  $s_1$  receives in case (ii), which is the probability that the a packet transmitted by  $s_1$  is successfully decoded either at  $s_2$  or  $d$ , is given by  $1 - (1 - q_{1|1}^{(s_2)} p_a)(1 - q_{1|1}^{(d)})$ .

Hence, the average service rate seen by  $s_1$  is

$$\mu_1 = (q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1 - p) + q_{1|1,2}^{(d)} p.$$

Using Loynes' theorem, queue  $Q_1$  will be stable if  $\lambda_1 < \mu_1$ .

### B. Analysis at the Secondary Node $s_2$

The average arrival rate at  $s_2$  is given by the sum of the exogenous arrivals with a rate  $\lambda_2$  as well as the endogenous arrivals from  $s_1$  with a rate of  $\lambda_{1 \rightarrow 2}$ , which remains to be calculated.

Packets from node  $s_1$  will arrive at  $s_2$  if the following events happen together:

- The queue  $Q_1 \neq 0$  and therefore  $s_1$  transmits; this happens with probability  $\lambda_1/\mu_1$ , as  $Q_1$  is a discrete-time M/M/1 queue.
- Node  $s_2$  is silent; this happens with probability  $(1 - p)$  in the dominant system  $\mathcal{S}'$ .
- Packets transmitted by  $s_1$  are unsuccessful at the destination  $d_1$  but are decoded successfully by  $s_2$  (this also implies that the flow controller allowed the packet to reach  $s_2$ ); this happens with probability  $(1 - q_{1|1}^{(d)}) q_{1|1}^{(s_2)} p_a$ .

The total arrival rate at  $s_2$  is therefore given by

$$\lambda_{s_2} = \lambda_2 + \lambda_{1 \rightarrow 2} = \lambda_2 + \mathbf{P}[Q_1 \neq 0](1 - p)(1 - q_{1|1}^{(d)}) q_{1|1}^{(s_2)} p_a$$

In order to compute the service rate at  $s_2$ , we need to analyze its departure process in the dominant system  $\mathcal{S}'$ , which results in one of two possible actions: either (a)  $Q_1 \neq 0$  while  $s_2$  transmits, in which case  $s_2$  transmits with probability  $p$ ; (using the fact that  $s_1$ 's queue in  $\mathcal{S}'$  is a discrete-time M/M/1 queue,  $Q_1 = 0$  with probability of  $(1 - \lambda_1/\mu_1)$ ), or (b)  $Q_1 = 0$  while  $s_2$  transmits, in which case  $s_2$  transmits with probability 1. When  $Q_1 = 0$ , node  $s_2$  can achieve a reception probability of  $q_{2|2}^{(d)}$  for its transmissions, while that reception probability reduces to  $q_{2|1,2}^{(d)}$  when  $Q_1 \neq 0$ . The average service rate of  $s_2$  is therefore

$$\begin{aligned} \mu_2 &= q_{2|2}^{(d)} \mathbf{P}[Q_1 = 0] + q_{2|1,2}^{(d)} \mathbf{P}[Q_1 \neq 0] p \\ &= \left(1 - \frac{\lambda_1}{\mu_1}\right) q_{2|2}^{(d)} + \frac{\lambda_1}{\mu_1} q_{2|1,2}^{(d)} p. \end{aligned}$$

Using Loynes' theorem, the queue at  $s_2$  will be stable if and only if  $\lambda_2 < \mu_2$ , and the cognitive network is stable if queues at both  $s_1$  and  $s_2$  are stable. Therefore, after some simple algebra, the stability conditions for the cognitive network for a fixed scheduling probability  $p$  can be written as

$$\lambda_1 < (q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1 - p) + q_{1|1,2}^{(d)} p, \quad (1)$$

$$\lambda_2 + \frac{(1 - p)(1 - q_{1|1}^{(d)}) q_{1|1}^{(s_2)} p_a - (q_{2|1,2}^{(d)} p - q_{2|2}^{(d)})}{(q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1 - p) + q_{1|1,2}^{(d)} p} \lambda_1 < q_{2|2}^{(d)}. \quad (2)$$

Based on the construction of the dominant system  $\mathcal{S}'$ , it is easy to see that the queue sizes of the dominant system, for each sample path realization, are never less than those of the original system, provided they are both initialized identically. This is because, in the dominant system,  $s_2$  transmits dummy packets even if it

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does not have any packets of its own, and therefore interferes with  $s_1$  in all cases that it would in the original system. Therefore, given  $\lambda_1 < \mu_1$ , if for some  $\lambda_2$ , the queue at  $s_2$  is stable in the dominant system, then the corresponding queue in the original system must be stable; conversely, if for some  $\lambda_2$  in the dominant system, the node  $s_2$  saturates (i.e., it always has a non-empty queue), then it will always have real packets to transmit, and as long as  $s_2$  has a packet to transmit, the behavior of the dominant system is identical to that of the original system (that is, if the dominant system is unstable, so is the original system). Therefore, we can conclude that the original system and the dominant system are indistinguishable at the boundary points. This is essentially the stochastic dominance argument presented in [7], and is applicable here as well.

Next, in order to understand the impact of the flow controller on the cognitive cooperative system  $\mathcal{S}$ , we have to find the value of  $p_a$  that maximizes  $\lambda_2$ . For this, we utilize the constrained optimization technique similar to that used in [2], [5] i.e., we fix  $\lambda_1$  and maximize  $\lambda_2$  as  $p$  varies over  $[0, 1]$ . First, we replace  $\lambda_1$  by  $x$  and  $\lambda_2$  by  $y$ . The boundary of the stability region for fixed  $p_a$  can now be written as,

$$y = q_{2|2}^{(d)} - \frac{(1-p)(1 - q_{1|1}^{(d)})q_{1|1}^{(s_2)} p_a - (q_{2|1,2}^{(d)} p - q_{2|2}^{(d)})}{(q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1-p) + q_{1|1,2}^{(d)} p} x, \quad (3)$$

$$\text{for } 0 \leq x \leq (q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1-p) + q_{1|1,2}^{(d)} p. \quad (4)$$

In order to maximize  $y$  for a fixed value of  $x$ , we need to understand the relationship between  $y$  and  $p_a$ . Differentiating  $y$  with respect to  $p_a$  gives

$$\frac{dy}{dp_a} = \frac{(1-p)(1 - q_{1|1}^{(d)})q_{1|1}^{(s_2)} p_a [p(q_{1|1,2}^{(d)} - q_{2|1,2}^{(d)} + q_{1|1}^{(d)}) - (q_{1|1}^{(d)} - q_{2|2}^{(d)})]}{(q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1-p) + q_{1|1,2}^{(d)} p} x; \quad (5)$$

The following observations can be made from (5):

- If  $p > \frac{q_{1|1}^{(d)} - q_{2|2}^{(d)}}{q_{1|1,2}^{(d)} - q_{2|1,2}^{(d)} + q_{1|1}^{(d)}}$  then  $\frac{dy}{dp_a} > 0$ , which means that  $y$  is an increasing function of  $p_a$ , therefore,  $p_a^* = 1$ . The stability region for this case is given by

$$\mathcal{R}_2 = \left\{ (\lambda_1, \lambda_2) : \frac{(1-p)(1 - q_{1|1}^{(d)})q_{1|1}^{(s_2)} - (q_{2|1,2}^{(d)} p - q_{2|2}^{(d)})}{(q_{1|1}^{(d)} + q_{1|1}^{(s_2)} - q_{1|1}^{(d)} q_{1|1}^{(s_2)})(1-p) + q_{1|1,2}^{(d)} p} \lambda_1 + \lambda_2 < q_{2|2}^{(d)}, \right. \\ \left. \text{for } 0 \leq \lambda_1 < (q_{1|1}^{(d)} + q_{1|1}^{(s_2)} - q_{1|1}^{(d)} q_{1|1}^{(s_2)})(1-p) + q_{1|1,2}^{(d)} p \right\}. \quad (6)$$

This is the case in which the secondary user provides complete cooperation to the primary user.

- If  $p < \frac{q_{1|1}^{(d)} - q_{2|2}^{(d)}}{q_{1|1,2}^{(d)} - q_{2|1,2}^{(d)} + q_{1|1}^{(d)}}$  then  $\frac{dy}{dp_a} < 0$ , which means that  $y$  is an decreasing function of  $p_a$ , therefore,  $p_a^* = 0$ . By substituting  $p_a^* = 0$  in (1) and (2), the stability region for this case is given by

$$\mathcal{R}_1 = \left\{ (\lambda_1, \lambda_2) : \frac{(q_{2|2}^{(d)} - q_{2|1,2}^{(d)} p)}{(q_{1|1}^{(d)})(1-p) + q_{1|1,2}^{(d)} p} \lambda_1 + \lambda_2 < q_{2|2}^{(d)}, \text{ for } 0 \leq \lambda_1 < (q_{1|1}^{(d)})(1-p) + q_{1|1,2}^{(d)} p \right\}. \quad (7)$$

This is equivalent to the case of no cooperation between the primary and the secondary users.

- For  $x < \{(q_{1|1}^{(d)} + q_{1|1}^{(s_2)} p_a - q_{1|1}^{(d)} q_{1|1}^{(s_2)} p_a)(1-p) + q_{1|1,2}^{(d)} p\}$  but  $x > \{(q_{1|1}^{(d)})(1-p) + q_{1|1,2}^{(d)} p\}$ , it follows from (1) that

$$p_a^* = \frac{x - q_{1|1}^{(d)}(1-p) - q_{1|1,2}^{(d)} p}{q_{1|1}^{(s_2)}(1 - q_{1|1}^{(d)})(1-p)}. \quad (8)$$

By substituting  $p_a^*$  into (2), the stability region for this case can be found to be

$$\mathcal{R}_3 = \left\{ (\lambda_1, \lambda_2) : \lambda_1 + \lambda_2 < q_{1|1}^{(d)} + q_{2|2}^{(d)} + (q_{1|1,2}^{(d)} + q_{2|1,2}^{(d)} - q_{1|1}^{(d)} - q_{2|2}^{(d)})p, \right. \\ \left. \text{for } q_{1|1}^{(d)}(1-p) + q_{1|1,2}^{(d)}p \leq \lambda_1 \leq (q_{1|1}^{(d)} + q_{1|1}^{(s_2)} - q_{1|1}^{(d)}q_{1|1}^{(s_2)})(1-p) + q_{1|1,2}^{(d)}p \right\}. \quad (9)$$

This is the case in which the secondary user provides partial cooperation to the primary user.

By allowing for this flexibility in terms of partial cooperation, it can be clearly seen that the stable throughput region of this system is at least as large, and in many cases larger than the stable throughput region that is achieved when the secondary user provides either full cooperation to the primary user, or no cooperation at all.

#### IV. CONCLUSION

This paper characterizes the stable-throughput region of a two user cognitive shared channel, in which we introduced the notion of a network level flow controller that controls the amount of cooperation provided to the primary user from the secondary user. We assume that the receivers are equipped with MPR capability and characterize the complete stability region for such a cognitive cooperative system by utilizing the idea of stochastic dominance. We characterize the optimal operation of the flow controller and identify situations under which the cognitive system transforms into one with no cooperation and full cooperation besides the case of partial cooperation from the secondary user to the primary user. We also observe that such flexibility in cooperation enabled by the flow controller results in a stable throughput region that is larger than or at least equal to the system without the flow controller.

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